Coupling Characteristics of Planar Dielectric Waveguides of Rectangular Cross Section

TRANG TRINH, STUDENT MEMBER, IEEE, AND RAJ MITTRA, FELLOW, IEEE

Abstract—**An approximate analytical method based on experimental results for predicting the coupling characteristics of various coupling structures is described. Expressions for the propagation constants were derived using the generalized effective dielectric constant method. For nonsymmetric coupling structures, the theoretical coupling coefficients were modified by a correction factor. Comparisons between experimental and theoretical results are presented.**

I. Introduction

A TYPICAL millimeter-wave integrated circuit that employs planar dielectrics as guiding media usually contains of several directional couplers and dielectric ring resonators [1], [2]. It is easy to recognize that the resonators consist of several directional couplers themselves. Part of the band-reject filter, for instance, is a nonsymmetric coupler, viz., two symmetric couplers and a nonparallel symmetric coupler. Hence, apart from the active devices, the dielectric directional couplers play an extremely important role in a millimeter-wave integrated circuit. For this reason, it is necessary to study the coupling characteristics of these couplers in great detail.

The coupling characteristics of the dielectric waveguides have been studied by several authors [3]–[6]. All of the coupling configurations in these studies were symmetrical in nature. In this paper, the scattering coefficients of nonsymmetric couplers are also investigated. Only the dielectric waveguide structure is discussed here although the same analysis can be directly applied to other guiding structures of rectangular cross section such as image guides, inverted strips, or insular guides.

II. Parallel Directional Couplers

A simple parallel directional coupler of length l is shown in Fig. 1. The port numbers are defined by the usual convention. The symmetry about the $x=0$ plane suggests that the propagating modes of the coupled structure are either symmetric (k_{even}) or antisymmetric (k_{odd}). These two wavenumbers k_{even} and k_{odd} are approximately given as [4]:

$$\left.\begin{matrix} k_{\text{even}} \\ k_{\text{odd}} \end{matrix}\right\} = k_z \left\{ 1 \pm 2 \frac{k_x^2 \xi \exp(-d/\xi)}{k_z^2 a (1 + k_x^2 \xi^2)} \right\} \tag{1}$$

where k_x and k_z are the transverse and longitudinal propagation constants of a single guide, respectively, and can be

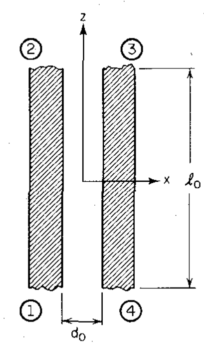

Fig. 1. Parallel coupling structure.

derived using the generalized effective dielectric constant method (see Appendix A); d is the spacing between the two guides; and ξ is the field decay coefficient

$$\xi = \left((\epsilon_{re}(y) - 1) k_0^2 - k_x^2 \right)^{-1/2}. \tag{2}$$

In expression (2), the relative dielectric constant of the material has been replaced by the effective dielectric constant which is given as

$$\epsilon_{re}(y) = \epsilon_r - (k_y/k_0)^2. \tag{3}$$

k_y is also obtained by the generalized effective dielectric constant method. A general approach for calculating the propagation constants of a coupled structure of two dielectric guides of different geometries is given by Arnaud [15].

Since, in a coupled structure, it is the interaction of the symmetric and the antisymmetric modes that induces coupling between the two dielectric guides [4], the scattering coefficients for the coupling section can be expressed as [16], [17]

$$|S_{21}| = \left| \cos \frac{k_{\text{even}} - k_{\text{odd}}}{2} l \right| \tag{4}$$

$$|S_{31}| = \left| \sin \frac{k_{\text{even}} - k_{\text{odd}}}{2} l \right| \tag{5}$$

where l is the total coupling length of the coupling section, and $(k_{\text{even}} - k_{\text{odd}})/2$ is defined as the coupling coefficient. It is clear that the larger the coupling coefficient, the stronger the degree of coupling between the two guides.

III. Nonparallel Coupling Structures

A. Symmetric Couplers

However, in practice, a more popular configuration is the symmetric coupler with nonuniform coupling spacing,

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The authors are with the Department of Electrical Engineering and the Coordinated Science Laboratory, University of Illinois, Urbana, IL 61801.

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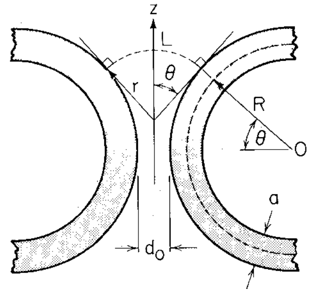


Fig. 2. Nonparallel symmetric coupler.

as shown in Fig. 2. In a dielectric waveguide, the equiphase fronts of the propagating modes are normal to the axial propagation direction. We assume that, with the existence of the second dielectric waveguide, these fronts can be approximated by cylindrical planes [2], [7]. Consequently, the separation between the incremental coupling lengths of the two lines is given by the arclength L. The total coupling of the two lines is the summation of the coupling from these incremental coupling lengths. The spacing d in (1) is replaced by L, which for the symmetric structure of Fig. 2, is given by

$$L=2r\theta \tag{6}$$

where θ is the angle from the incremental coupling length to the center line, and r is the radii of the cylindrical phase fronts which can be expressed as a function of θ as

$$r=\frac{d_0/2+(R+a/2)(1-\cos\theta)}{\sin\theta} \tag{7}$$

where d_0 is the smallest spacing between two curved guides.

The scattering coefficients can be derived by substituting (6), (7) and (1) into (4) and (5) and using $L=Rd\theta$ to get

$$|S_{21}|=|\cos(KI_s)| \tag{8}$$

and

$$|S_{31}|=|\sin(KI_s)| \tag{9}$$

where

$$K=\frac{4k_x^2\xi R}{k_z a(1+k_x^2\xi^2)} \tag{10}$$

$$I_s=\int_0^{\pi/2}\exp\left[-\frac{\theta\{d_0+2(R+a/2)(1-\cos\theta)\}}{\xi\sin\theta}\right]d\theta. \tag{11}$$

In this equation, the integration has been used to represent the summation of the couplings of all incremental coupling lengths. Also, we have assumed that, for the radius R sufficiently large, the wavenumbers of both guides can be roughly equal to that of a straight guide. The experimental and computed results of $|S_{21}|$ and $|S_{31}|$ as a function of the spacing d_0 are plotted together, as shown in Figs. 3(a) and (b). The guide dimensions are 2.27×1.0 cm; $\epsilon_r=2.6$.

B. Nonsymmetric Couplers

Another very important class of dielectric couplers is the nonsymmetric structure shown in Fig. 4 where ports 1 and 2 are connected by a straight dielectric guide, and 3 and 4 by a curved dielectric guide. The energy is excited at port 1. Using the previous assumptions, i.e., the wavefronts are normal to the axial propagation direction and these fronts are cylindrical planes, the separation between the incremental coupling lengths of a nonsymmetric coupler becomes

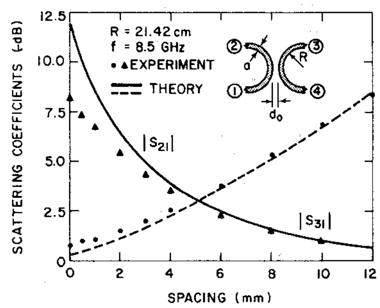
$$L=r\theta \tag{12}$$

where

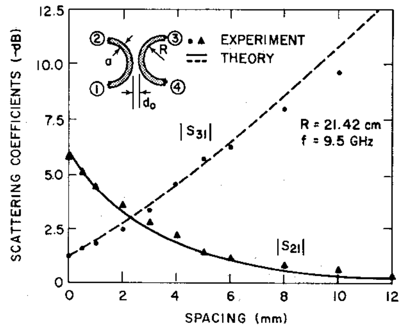
$$r=\frac{d_0+(R+a/2)(1-\cos\theta)}{\sin\theta}. \tag{13}$$

The integral expression for the summation of the couplings of incrementals lengths for this structure can be expressed as

$$I_n=\int_0^{\pi/2}\exp\left[-\frac{\theta\{d_0+(R+a/2)(1-\cos\theta)\}}{\xi\sin\theta}\right]d\theta. \tag{14}$$



(a)



(b)

Fig. 3. Scattering coefficients of a symmetric coupler versus guide spacing d_0 at: (a) $f=8.5$ GHz; (b) $f=9.5$ GHz.

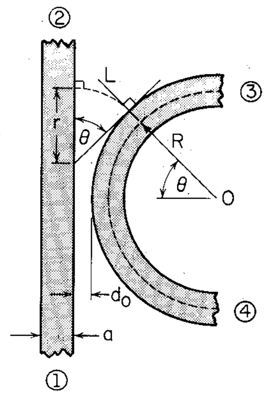


Fig. 4. Nonsymmetric coupler.

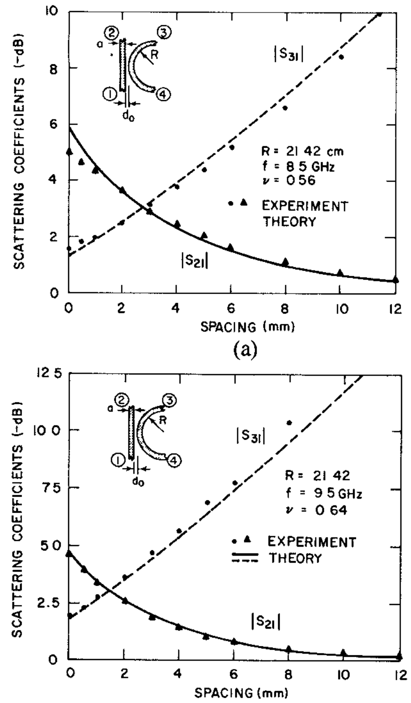


Fig. 5. Scattering coefficients of a nonsymmetric coupler versus guide spacing d_0 at (a) $f=8.56$ Hz; (b) $f=8.56$ Hz.

TABLE I
TYPICAL VALUES OF THE CORRECTION FACTOR ν FOR NONSYMMETRIC COUPLERS

Radius (cm)	Frequency (GHz)	Correction Factor
18.9 (R/a = 8.3)	8.5	0.58
	9.0	0.63
	9.5	0.64
	10.0	0.69
21.42 (R/a = 9.43)	8.5	0.56
	9.0	0.61
	9.5	0.64
	10.0	0.68
23.96 (R/a = 10.55)	8.5	0.55
	9.0	0.61
	9.5	0.64
	10.0	0.68

Up to this point, the coupling coefficient for the nonsymmetric coupler is derived using the assumption that the propagation constants of the curved and the straight dielectric guides are identical. For most practical cases, this assumption is not valid. The difference in the propagation constants of the two guides will inevitably degrade the coupling coefficient which so far is assumed to be ideal. It was found that the experimental values of the coupling coefficient were always much less than the theoretical values. Hence, it is necessary to multiply the derived coefficient by a correction factor ν, and the scattering coefficients for a nonsymmetric coupler become

$$|S_{21}|=|\cos(\nu K I_n)| \tag{15}$$

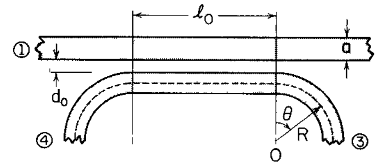


Fig. 6. Coupling structure incorporating a straight section and nonsymmetric curved arms.

and

$$|S_{31}|=|\sin(\nu K I_n)| \tag{16}$$

where ν is determined experimentally and, in general, depends on the operating frequency and the curvature of the curved guide. This correction factor is designed to take into account the differences between the approximated propagation constants of the two guides, radiation from the bend, mode conversion in the curved guides, junction discontinuities, etc., which were not included in the simple theoretical model for calculating the scattering coefficients derived in this section. For any given frequency, only one simple experiment is necessary to determine ν (usually on S_{21} since it is easy to measure). Once obtained, the same value can be used to calculate the scattering coefficients for any other separations of the guides. For most practical cases, ν was found to vary in the range from 0.5 to 0.7. Table I shows some typical values of ν for different frequencies and radii of curvature. Fig. 5(a) and (b) compare the experimental and calculated results for the scattering coefficients at some frequencies as a function of the guide spacing d_0.

C. Coupling Structure Incorporating a Straight Coupling Section with Two Nonsymmetric Curved Arms

Sometimes it is desirable to design a coupling structure that is capable of transferring all energy from one guide to the other. The two structures described in the previous sections by themselves usually do not have that capability unless the radius of the curved guide becomes prohibitively large. The coupling can be greatly improved if a straight dielectric waveguide section is inserted between two curved connecting arms. The structure with symmetric connecting arms has been investigated elsewhere [6] and is not mentioned again.

The coupling structure in which a straight guide section l_0 is inserted between two nonsymmetric curved dielectric guides is shown in Fig. 6. Port 1 to 2 is a straight guide; from 4 to 3 is a straight guide inserted between two curved dielectric guides. Assuming the reflection at the junctions between the curved and the straight guides is small, and since the fields at these junctions are continuous, the total coupling of the structure is the sum of the coupling from the straight section and that from the curved connecting arms. Consequently, the scattering coefficients for this structure become

$$|S_{21}|=\left|\cos\nu\left(\frac{k_{\text{even}}(s)-k_{\text{odd}}(s)}{2}l_0+\frac{k_{\text{even}}(c)-k_{\text{odd}}(c)}{2}l\right)\right| \tag{17}$$

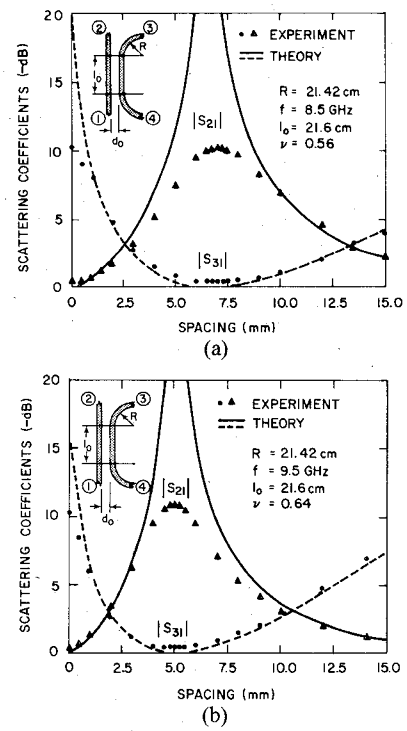


Fig. 7. Scattering coefficients of the coupling structure shown in Fig. 6 versus guide spacing d_0 at (a) $f=8.5$ GHz; (b) $f=9.56$ Hz.

and

$$|S_{31}|=\left(1-|S_{21}|^2\right)^{1/2} \tag{18}$$

where ν is the same correction factor that was obtained for the nonsymmetric coupler (see Section III-B). In (17), the first term inside the argument is for the uniform spacing section, and the second term is for the nonsymmetric connecting arms.

If k_{even} and k_{odd} are substituted into (17), the scattering coefficients can be rewritten in the form

$$|S_{21}|=\left|\cos\left(\nu K\left\{\frac{\exp(-d_0/\xi)}{2R}l_0+I_n\right\}\right)\right| \tag{19}$$

where I_n is given in (14).

The experimental and calculated results of $|S_{21}|$ and $|S_{31}|$ as a function of guide spacing d_0 are plotted together and are shown in Figs. 7(a) and (b).

IV. Experimentation

Experiments were carried out in X-band because the component sizes are more manageable and the measurements are more accurate. To reduce the effect of the large mismatch caused by the launching devices, the fundamental E_{11}^y mode was launched from an improved rectangular horn [8]. The guiding media were fabricated from plexiglass ($\epsilon_r=2.6$). The guide dimensions are 2.27×1.00 cm. The curved section was sufficiently large since the purpose of this investigation is to study the coupling characteristics only. Each guide is supported by bubble styrofoam ($\epsilon_r\simeq1$). The entire structure is surrounded by absorbers to eliminate stray radiation.

It is interesting to note that for a symmetric coupler even though the separation between the two guides is larger than that of a nonsymmetric coupler, the coupling of the symmetric configuration is stronger than the latter one. Also, for a given radius of curvature, the coupling is decreased by increasing frequency.

For the coupling structure in which a straight guide section is inserted between two curved arms, the propagation constants of each guide in the straight section in general is not identical due to the interference of the radiation from the junctions and the guided modes [7]. For this reason, the actual coupling coefficient of this uniformly spaced section should be less than that of the simple parallel coupler described in Section II, and must be corrected by a factor ν in order to match the experimental and theoretical results. It is almost impossible to theoretically predict the exact value of ν. Hence, the correction factor will be determined experimentally. The measurements are calibrated by first separating the two guides sufficiently far apart such that they are virtually uncoupled. Next, we use (15) and (16) to determine ν by bringing the two guides closer together until $|S_{21}|$ and $|S_{31}|$ are close to the -3-dB level. In all experiments, if the energy is excited at port 1 of the straight guide, we found that the correction factor for this straight section assumed the same value as that for a nonsymmetric coupler. This is desirable since only a very simple measurement is made to obtain ν, and this value is used for all computations of both structures. For a given radius of curvature, Table I shows that higher values of ν, which imply stronger coupling, are obtained at higher frequencies. This is due to the fact that the effective coupling length (in terms of wavelengths) is longer at higher frequencies.

For a symmetric coupler with the dielectric waveguide as the guiding structure, the experimental values of ν are found to be approximately equal to 1. However, for other guiding structures, the correction factors for the coupling coefficient may be different from unity, and a measurement for ν may be necessary.

Scattering coefficients of various coupling structures were measured and compared with calculated results. Both agreed well throughout. The scattering coefficient $|S_{41}|$ has been ignored in all figures since it has less than -20 dB for all measurements.

V. Conclusions

Coupling characteristics of various structures were investigated experimentally and theoretically. For a nonsymmetric coupler, it is only necessary to make one simple experiment to determine the correction factor for the coefficient. Once obtained, this value can be used to calculate the coupling of the structure at any arbitrary spacing. Comparisons of calculated and measured scattering coefficients are very good. This analysis can also be directly applied to other guiding structures of rectangular cross-section where the effective dielectric constant method can be applied. For the coupling structure for the two guides of different geometries, the approach described in this paper is still valid as long as the propagation constants of each guide are approximately the same. However, all the formulas for calculating $|S_{21}|$ and $|S_{31}|$ for the latter structure will be entirely different from those given in this paper.

Appendix A
Generalized Effective Dielectric Constant Method

In the past, several authors have attempted to find the exact solutions for the propagation constants of a dielectric waveguide using the infinite system of linear equations [9], [10]. For many applications, these approaches are impractical since they needlessly require too much computation. Unless a very large number of terms are used, the results can be doubtful.

A simple analytical approach for calculating the propagation constants of a dielectric waveguide was introduced by Marcatili [4]. However, this approximation failed to predict the propagation of the fundamental mode at low frequencies. The effective dielectric constant method was considered a slight modification to the Marcatili's approach [11]. Here we introduce the generalized effective dielectric constant method. Before describing this method, it is helpful to give a brief review of the effective dielectric constant method which has been studied extensively elsewhere [12].

In general, the effective dielectric constant $\epsilon_{re}(y)$ is defined as

$$\epsilon_{re}(y) = (k_z/k_0)^2 = \epsilon_r - (k_y/k_0)^2 \qquad \text{(A-1)}$$

where ϵ_r is the relative permittivity of the dielectric material; k_y is the transverse propagation constant of a dielectric slab of thickness b; and k_0 is the free-space wave number.

The axial propagation constant of the original guide is given by

$$k_z^2 = \epsilon_{re}(y)k_0^2 - k_x^2 \qquad \text{(A-2)}$$

where k_x is the transverse propagation constant of a dielectric slab of thickness a and permittivity $\epsilon_{re}(y)$; and k_x satisfies

$$\epsilon_{re}(y)k_0^2 - k_x^2 = k_0^2 + 1/\xi^2 \qquad \text{(A-3)}$$

where

$$\xi = \left((\epsilon_{re}(y)-1)k_0^2 - k_x^2\right)^{-1/2}. \qquad \text{(A-4)}$$

ξ is the decay coefficient in the x-direction in the air medium.

In short, k_x was obtained from k_y through the effective dielectric constant $\epsilon_{re}(y)$.

Obviously, this sequence can be reversed by first solving k_x for a dielectric slab of thickness a and relative permittivity ϵ_r. The new effective dielectric constant is

$$\epsilon_{re}(x) = \epsilon_r - (k_x/k_0)^2. \qquad \text{(A-5)}$$

The axial propagation constant for this case is obtained by

$$k_z^2 = \epsilon_{re}(x)k_0^2 - k_y^2 = k_0^2 + 1/\eta^2 \qquad \text{(A-6)}$$

where k_y is the transverse propagation constant of a dielectric slab of thickness b and relative dielectric constant $\epsilon_{re}(x)$; and

$$\eta = \left((\epsilon_{re}(x)-1)k_0^2 - k_y^2\right)^{-1/2} \qquad \text{(A-7)}$$

η is the field decay coefficient in the y-direction in the air medium.

Hence, for this sequence, k_y is related to k_x through the effective dielectric constant $\epsilon_{re}(x)$. The new effective permittivity just described is believed to be equivalent to the effective permeability defined by other authors [13], [14].

Since each approach has its own merits, and k_x and k_y are related through either $\epsilon_{re}(x)$ or $\epsilon_{re}(y)$, it may be desirable to combine these two approaches to obtain a "generalized" effective dielectric constant method. To do so, the dielectric waveguide is first converted into an infinite slab in x to obtain k_{y1}. k_{x1} is then obtained from k_{y1} using $\epsilon_{re}(y)$. So far this procedure is the same as the original effective dielectric constant method described by Knox and Toulios. However, a new value k_{y2} is obtained from k_{x1} using $\epsilon_{re}(x)$ as described from (A-5) to (A-7). Then k_{x2} is obtained from k_{y2} using $\epsilon_{re}(y)$ again, and so on. After a few iterations, the transverse propagation constants k_{xn} and k_{yn} will converge, and from which the final longitudinal propagation constant k_z is derived. Some experimental and theoretical results for this method were presented in [13].

In Section III, all the propagation constants and field decay coefficients were obtained by this generalized effective dielectric constant method.

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Trang Trinh (S'77) was born in Nha Trang, Vietnam, on November 28, 1956. He received the B.S. degree in electrical engineering from Wilkes College, Wilkes Barre, PA, in 1978 and the M.S. degree in electrical engineering from the University of Illinois, Urbana, IL, in 1980. He is currently studying toward the Ph.D. degree at the University of Illinois, where he has been working on the millimeter-wave integrated circuits and components.

Mr. Trinh is a member of Sigma Xi.

Raj Mittra (S'54–M'57–SM'69–F'71) is Professor of Electrical Engineering and Associate Director of the Electromagnetic Laboratory of the University of Illinois. He serves as a Consultant to several industrial and governmental organizations, including the NASA Jet Propulsion Laboratory of the California Institute of Technology. A paper by him in the September issue of IEEE/AP-S has won the 1978 "Best Paper Award" of the Antenna and Propagation Society.

He is past president of the IEEE antennas Propagation Society and an active contributor to many activities of the society.